

1 a. \_\_\_\_\_  
 5ε \_\_\_\_\_  
 4ε \_\_\_\_\_  
 3ε \_\_\_\_\_  
 2ε \_\_\_\_\_  
 ε \_\_\_\_\_  
 0 \_\_\_\_\_

$$N = 4 \text{ particles}$$

$$U = 5ε \text{ energy}$$

Distribution.		0	ε	2ε	3ε	4ε	5ε	(ii) No microstates ↓
(i)	1	3	0	0	0	0	1	4
	2	2	1	0	0	1	0	12
	3	2	0	1	1	0	0	12
	4	1	2	0	1	0	0	12
	5	1	1	2	0	0	0	12
	6	0	3	1	0	0	0	4
								Total = 56

Number of microstates in total = 56.

Mean occupation:

$$(ii) \frac{1}{56} \{ 4 \times 3 + 12 \times 2 + 12 \times 2 + 12 \times 1 + 12 \times 1 + 4 \times 0 \} = \frac{84}{56} = 1.50$$

$$n_1 = \frac{1}{56} \{ 4 \times 0 + 12 \times 1 + 12 \times 0 + 12 \times 2 + 12 \times 1 + 4 \times 3 \} = \frac{60}{56} = 1.07$$

$$n_2 = \frac{1}{56} \{ 4 \times 0 + 12 \times 0 + 12 \times 1 + 12 \times 0 + 12 \times 2 + 4 \} = \frac{46}{56} = 0.82$$

$$n_3 = \frac{1}{56} \{ 4 \times 0 + 12 \times 0 + 12 \times 1 + 12 \times 1, \dots \} = \frac{24}{56} = 0.43$$

$$n_4 = \frac{1}{56} \{ 12 \} = \frac{12}{56} = 0.21$$

$$n_5 = \frac{1}{56} \times 4 = \frac{4}{56} = 0.07$$

1(b). N atoms at temp T in level scheme shown

$2\epsilon$  \_\_\_\_\_

$\epsilon$  \_\_\_\_\_

0 \_\_\_\_\_

(i) Partition function 
$$Z = \sum_j \exp(-\epsilon_j/kT)$$
  

$$= 1 + \exp(-\epsilon/kT) + \exp(-2\epsilon/kT) \quad 2$$

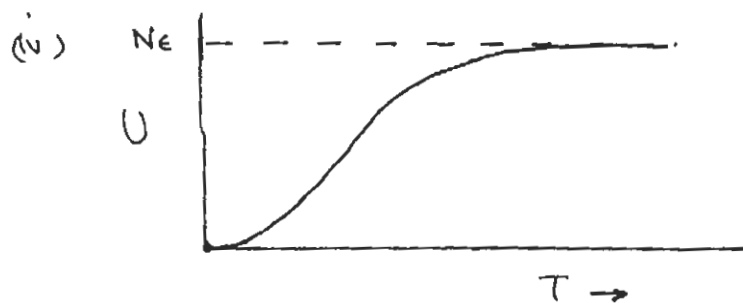
(ii) 
$$U = NkT^2 \frac{d}{dT} \left\{ \ln [1 + \exp(-\epsilon/kT) + \exp(-2\epsilon/kT)] \right\}$$
  

$$= NkT^2 \cdot \left\{ \frac{(\epsilon/kT^2) \exp(-\epsilon/kT) + (2\epsilon/kT^2) \exp(-2\epsilon/kT)}{[1 + \exp(-\epsilon/kT) + \exp(-2\epsilon/kT)]} \right\}$$
  

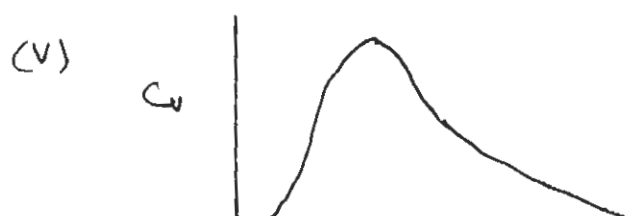
$$= NkT^2 \cdot \frac{(\epsilon/kT^2) \exp(-\epsilon/kT) [1 + 2\exp(-\epsilon/kT)]}{[1 + \exp(-\epsilon/kT) + \exp(-2\epsilon/kT)]}$$
  

$$= \frac{N\epsilon \cdot \exp(-\epsilon/kT) [1 + 2\exp(-\epsilon/kT)]}{[1 + \exp(-\epsilon/kT) + \exp(-2\epsilon/kT)]} \quad 2$$

(iii) As  $T \rightarrow 0$   $\exp(-\epsilon/kT) \rightarrow 0$   $U \rightarrow 0$   
 As  $T \rightarrow \infty$   $\exp(-\epsilon/kT) \rightarrow 1$   $U \rightarrow \frac{N\epsilon \cdot 1.3}{3} \rightarrow N\epsilon \quad 2$



Sketch from above values.

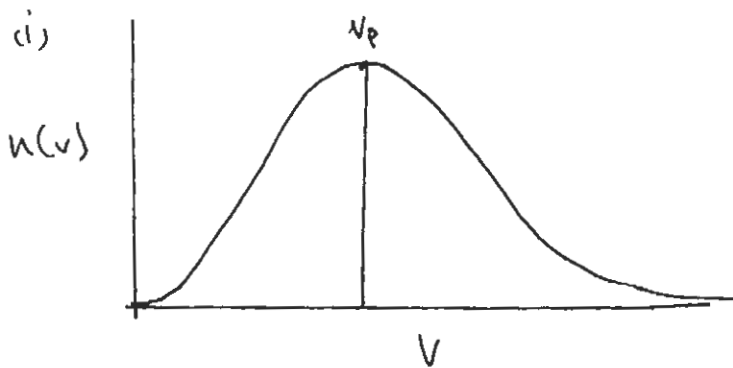


Sketch from slope of  $U$  vs  $T$  curve

[10]

1c.

$$n(v) = 4\pi N \left( \frac{m}{2\pi kT} \right)^{3/2} v^2 \exp\left(-\frac{mv^2}{2kT}\right).$$



2

(ii) Most probable velocity  $v_p$

$$\frac{dn}{dv} = 0 = 4\pi N \left( \frac{m}{2\pi kT} \right)^{3/2} \left\{ v^2 \cdot \frac{-2mv}{2kT} \exp\left(-\frac{mv^2}{2kT}\right) + 2v \exp\left(-\frac{mv^2}{2kT}\right) \right\}$$

$$0 = -\frac{2mv^3}{2kT} + 2v$$

$$v_p^2 = \frac{2kT}{m}$$

$$v_p = \left( \frac{2kT}{m} \right)^{1/2} \quad 2$$

(iii)

$$\overline{v_m^2} = \frac{\int_0^\infty 4\pi N \left( \frac{m}{2\pi kT} \right)^{3/2} v^4 \exp\left(-\frac{mv^2}{2kT}\right) dv}{\int_0^\infty 4\pi N \left( \frac{m}{2\pi kT} \right)^{3/2} v^2 \exp\left(-\frac{mv^2}{2kT}\right) dv} = \frac{I_4}{I_2}$$

$$= \frac{3}{26} \cdot \frac{I_2}{I_2} = \frac{3}{2} \cdot \frac{2kT}{m} = \frac{3kT}{m} \quad 2$$

$$b = \frac{m}{2kT}.$$

(iv)

$$U_{mole} = \frac{1}{2} m \overline{v_m^2} \cdot N_A = \frac{m}{2} \cdot \frac{3kT}{m} \cdot N_A = 1.5 \times 1.38 \times 10^{-23} \times 300 \times 6 \times 10^{23}$$

$$= 3726 \text{ J.}$$

2

[8]

1.d Diatomic molecules. H.D.

$$I=2 \quad \text{---} \quad \frac{E}{\frac{6h^2}{2\mu r^2}}$$

$$I=1 \quad \text{---} \quad \frac{2h^2}{2\mu r^2}$$

$$I=0 \quad \text{---} \quad 0$$

$E_R$   
Rotation.

$$n \quad \text{---} \quad E$$

$$4 \quad \text{---} \quad$$

$$3 \quad \text{---} \quad$$

$$2 \quad \text{---} \quad \frac{5}{2} h\nu$$

$$1 \quad \text{---} \quad \frac{3}{2} h\nu$$

$$0 \quad \text{---} \quad \frac{1}{2} h\nu$$

$E_V$   
Vibration.

(i)  $E_R = \frac{2h^2}{2\mu r^2}$  . Reduced mass  $\mu = \frac{m \cdot 2m}{m+2m} = \frac{2}{3}m$   
 $= 0.67 \times 1.66 \times 10^{-27} \text{ kg}.$

$$E_R = \left( \frac{6.63 \times 10^{-34}}{2\pi} \right)^2 \cdot \frac{1}{0.67 \times 1.66 \times 10^{-27} \times (1.05 \times 10^{-10})^2} = 9.08 \times 10^{-22} \text{ J} \left( 5.68 \times 10^{-3} \text{ eV} \right)$$

$$E_V = h \left( \frac{B}{\mu} \right)^{1/2} = \frac{6.63 \times 10^{-34}}{2\pi} \left( \frac{6.91 \times 10^2}{0.67 \times 1.66 \times 10^{-27}} \right)^{1/2} = 8.32 \times 10^{-20} \text{ J} \left( 0.52 \text{ eV} \right)$$

(ii) Rotational motion excited at  $\Theta_R$   
 where  $k\Theta_R = 9.08 \times 10^{-22}$

$$\Theta_R = \frac{9.08 \times 10^{-22}}{1.38 \times 10^{-23}} = 65.8 \text{ K}$$

Vibrational motion at  
 $k\Theta_V = 8.32 \times 10^{-20} \text{ J}$

$$\Theta_V = \frac{8.32 \times 10^{-20}}{1.38 \times 10^{-23}} = 6028 \text{ K}.$$

(iii) At  $T = 20 \text{ K}.$

Translational motion contributes  $\frac{3R}{2}$  to molar  $C_V$  →

Rot and vibrational motions not excited thus  $C_V \sim \frac{3R}{2}$  2

At  $T = 300 \text{ K}$

Translational and rotational motions excited  $C_V = \frac{3R}{2} + R \approx \frac{5R}{2}$

Vibrational motion not excited.

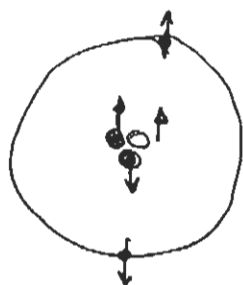
trans + rot 2

187

12.

Construct He<sup>3</sup> atom

(i)

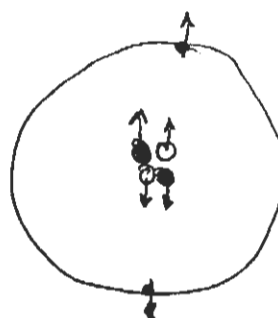


2 (1s) electrons spins  $\uparrow\downarrow$

2  $\uparrow$  protons spins  $\uparrow\downarrow$

1  $\uparrow$  neutron spin  $\uparrow$

He<sup>4</sup> atom



2 (1s) electrons spins  $\uparrow\downarrow$

2  $\uparrow$  protons spins  $\uparrow\downarrow$

2  $\uparrow$  neutrons spins  $\uparrow\downarrow$

(ii) Electronic angular momentum  $J=0$

Nuclear angular momentum  $I = \frac{1}{2}$

$$\text{Total } F = \underline{I} + \underline{J} = \underline{\frac{1}{2}}$$

Electronic angular momentum  $J=0$

Nuclear angular momentum  $I=0$

$$\text{Total } \underline{F} = \underline{I} + \underline{J} = 0.$$

(iii) Atoms with half integer angular momentum — like He<sup>3</sup> are fermions.  
Atoms with 0 or integer angular momentum (like He<sup>4</sup>) are bosons.

(iv) He<sup>3</sup> atom populations at low temp. ( $T=2K$ )



← up to Fermi energy  $\epsilon_F$ .

← each state has  $\uparrow\downarrow$  He<sup>3</sup> atoms only

(v) He<sup>4</sup> atom populations at low temp. ( $T=2K$ )



Small number He<sup>4</sup> in excited states

← very many condensed into ground state.

[60]

1f.

$$(iii) B_c(T) = B_c(0) \left[ 1 - \left( \frac{T}{T_c} \right)^2 \right]$$

$$0.05 = 0.080 \left[ 1 - \left( \frac{T}{7.2} \right)^2 \right]$$

$$0.625 = 1 - \left( \frac{T}{7.2} \right)^2$$

$$\left( \frac{T}{T_c} \right)^2 = 1 - 0.625 = 0.375$$

$$\frac{T}{T_c} = 0.612$$

$$T = 0.612 T_c = 0.612 \times 7.2 = 4.41 \text{ K.}$$

(iv) At  $T = 5.6 \text{ K}$  — calculate critical field

$$B_c(T) = 0.080 \left[ 1 - \left( \frac{5.6}{7.2} \right)^2 \right]$$

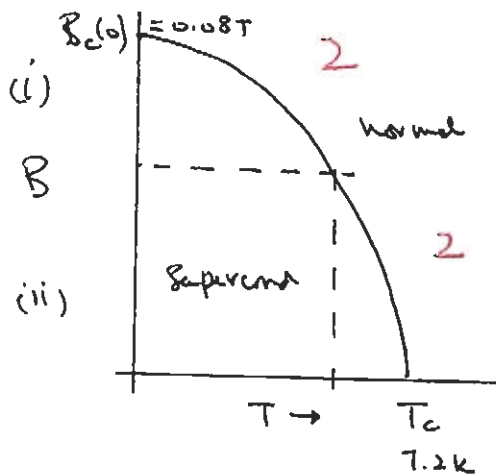
$$= 0.080 [1 - 0.605] = 0.080 \times 0.395$$

$$= 0.0316 \text{ T.}$$

Since  $B = 0.035 \text{ T}$  is greater than this

then at  $B = 0.035 \text{ T}$  and  $T = 5.6 \text{ K}$  — Pb is normal

[8]



19.

$$E = \pm \mu B = \pm 1.0 \times 9.27 \times 10^{-24} \times 1.5$$

$$= 1.391 \times 10^{-23} \text{ J}$$

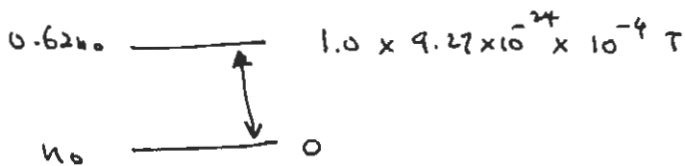
(i) Splitting  $= 2\mu B = 2.78 \times 10^{-23} \text{ J}.$

(ii) Population ratio  $\frac{n_1}{n_0} = \exp\left(-\frac{2\mu B}{kT}\right) \quad T = 4.2 \text{ K}.$

$$= \exp\left(-\frac{2.78 \times 10^{-23}}{1.38 \times 10^{-23} \times 4.2}\right)$$

$$= \exp(-0.480)$$

$$= 0.62.$$



$$0.62 = \exp\left(-\frac{9.27 \times 10^{-28}}{kT}\right)$$

$$\ln(0.62) = -\frac{9.27 \times 10^{-28}}{1.38 \times 10^{-23} \times T}$$

$$-0.48 = -\frac{9.27 \times 10^{-5}}{1.38 \times T} \quad T = \frac{9.27 \times 10^{-5}}{0.48 \times 1.38} = 1.4 \times 10^{-4} \text{ K}$$

2(a)

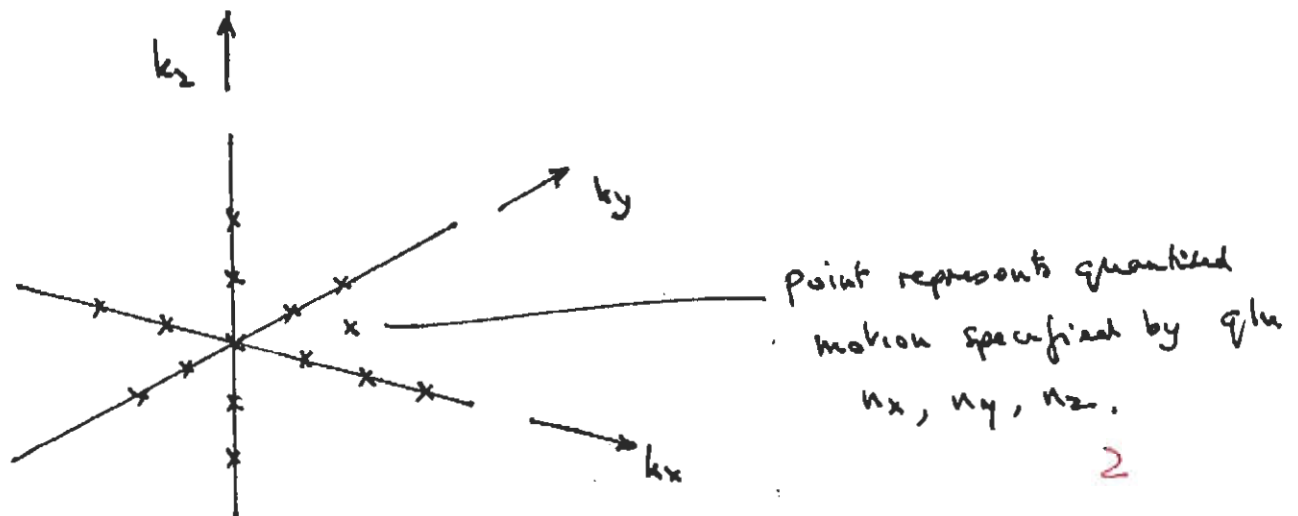
(i) For motion in a box of side  $L$

Using periodic boundary conditions.  $\exp(ik_x L) = 1$ .

Get quantised values

$$\left. \begin{aligned} k_x &= \frac{2\pi}{L} n_x \\ k_y &= \frac{2\pi}{L} n_y \\ k_z &= \frac{2\pi}{L} n_z \end{aligned} \right\} \begin{aligned} &\text{where } n_x, n_y, n_z \\ &\text{are } 0, \pm 1, \pm 2 \\ &\text{- integers.} \end{aligned}$$

(ii)



(iii) Number of states  $g(k) dk$  lying between  $k \rightarrow k+dk$

$$g(k) dk = \frac{\text{Volume in } k \text{ space}}{\left(\frac{2\pi}{L}\right)^3} \quad \text{Volume / mode}$$

Volume in  $k$  space is spherical shell - radius  $k$  and thickness  $dk$  - volume =  $4\pi k^2 dk$ .

Thus for spinless particles

$$g(k) dk = \frac{4\pi k^2 dk}{\left(\frac{2\pi}{L}\right)^3}$$

For electrons with 1/2 occupation of each point

$$g(k) dk = \frac{2 \cdot V \cdot 4\pi k^2 dk}{(2\pi)^3} \quad \text{where } V = L^3.$$



(iv) For free electron  $\epsilon = \frac{\hbar^2 k^2}{2m}$ .

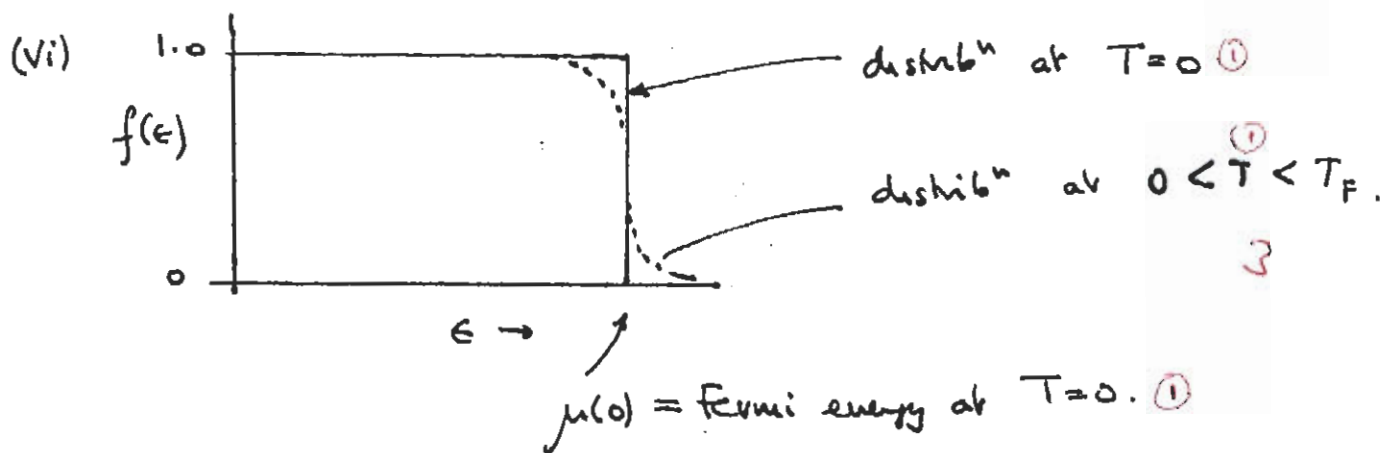
(v)  $k^2 = \frac{2m\epsilon}{\hbar^2}$  thus  $2k dk = \frac{2m}{\hbar^2} d\epsilon$  ①

$k = \left(\frac{2m\epsilon}{\hbar^2}\right)^{1/2}$  ①

Thus  $g(k) dk = \frac{2 \cdot V \cdot 4\pi k^2 \cdot dk}{(2\pi)^3}$

transforms to  $g(\epsilon) d\epsilon = \frac{2 \cdot V \cdot 4\pi}{(2\pi)^3} \cdot \left(\frac{2m\epsilon}{\hbar^2}\right)^{1/2} \cdot \frac{m}{\hbar^2} d\epsilon$

$g(\epsilon) d\epsilon = \frac{2V}{(2\pi)^2} \left(\frac{2m}{\hbar^2}\right)^{3/2} \epsilon^{1/2} d\epsilon$  ① 3



(vii) At  $T=0$  all states with  $k < k_F$  are filled with  $\uparrow\downarrow$  electrons

$N = \frac{4}{3}\pi k_F^3 \cdot 2 \cdot \frac{V}{(2\pi)^3}$  ①

$\swarrow$  vol  $k$  space  $\nearrow \uparrow\downarrow$   $\nwarrow$  1/vol/state ①

$\therefore k_F^3 = \frac{(2\pi)^3 \cdot 3N}{4\pi \cdot 2V} = \frac{3\pi^2 N}{V}$  ①

Fermi energy  $\mu(0) = \frac{\hbar^2 k_F^2}{2m} = \left(\frac{\hbar^2}{2m}\right) \cdot \left(\frac{3\pi^2 N}{V}\right)^{2/3}$  ① 4

(viii) Silver - molar volume =  $10.27 \times 10^{-6} \text{ m}^3$

$$\text{Thus } \frac{N}{V} = \frac{6 \times 10^{23}}{10.27 \times 10^{-6}} \quad (1)$$

$$\begin{aligned} \mu(0) &= \left( \frac{h^2}{2m} \right) \left( \frac{3\pi^2 \times 6 \times 10^{23}}{10.27 \times 10^{-6}} \right)^{2/3} \\ &= \left( \frac{6.63 \times 10^{-34}}{2\pi} \right)^2 \cdot \frac{1}{2 \times 9.11 \times 10^{-31}} \left( \frac{2\pi^2 \times 6 \times 10^{23}}{10.27 \times 10^{-6}} \right)^{2/3} \quad (1) \\ &= 9.01 \times 10^{-19} \text{ J} \quad (1) \quad (5.63 \text{ eV}). \\ &= 5.63 \text{ eV}. \quad 3 \end{aligned}$$

(ix)  $C_v$  at 5K.

$$\begin{aligned} C_v &= \left( \frac{\partial U}{\partial T} \right) \quad (1) = \frac{2N\pi^2 k^2 T}{4\mu(0)} \quad (1) \\ &= \frac{2 \times 6 \times 10^{23} \pi^2 (1.38 \times 10^{-23})^2 \times 5}{4 \times 9.01 \times 10^{-19}} = \\ &= 3.13 \times 10^{-3} \text{ J K}^{-1}. \quad (1) \quad 3 \end{aligned}$$

26.

(i) Radiation in equilibrium within a cavity at temperature  $T$  assumes a frequency distribution  $g(\nu)$  determined only by  $T$ . This is black body radiation.

(ii) Relate  $k$  and  $\nu$

$$k = \frac{2\pi}{\lambda}, \quad \nu = \frac{c}{\lambda} \quad \text{thus } k = \frac{2\pi\nu}{c}$$

$$dk = \frac{2\pi}{c} \cdot d\nu$$

$$\text{Thus from } g(k) dk = 2 \cdot V \cdot \frac{4\pi k^2 dk}{(2\pi)^3}$$

$$\begin{aligned} \text{Giv } g(\nu) d\nu &= 2 \cdot V \cdot \frac{4\pi}{(2\pi)^3} \cdot \frac{4\pi^2 \nu^2}{c^2} \cdot \frac{2\pi}{c} d\nu \\ &= 8V\pi \frac{\nu^2}{c^3} d\nu \end{aligned}$$

(iii)  $h\nu$  is photon energy

$[\exp(h\nu/kT) - 1]^{-1}$  is probability of state of energy  $h\nu$  is occupied - Bose occupation factor!

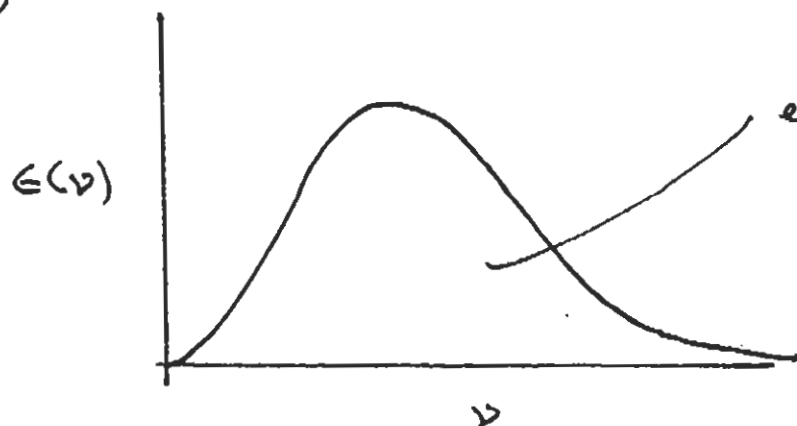
$$\text{(iv) As } \nu \rightarrow 0 \quad [\exp(h\nu/kT) - 1]^{-1} \rightarrow \frac{1}{[1 + h\nu/kT - 1]} \rightarrow \frac{kT}{h\nu}$$

thus  $\epsilon(\nu) \propto \nu^2 \rightarrow 0$  as  $\nu \rightarrow 0$ .

$$\text{As } \nu \rightarrow \infty \quad \exp(h\nu/kT) \rightarrow \infty \quad \epsilon(\nu) \rightarrow 0.$$

$\exp(h\nu/kT)$  in denominator outweighs  $\nu^2$  in numerator

(v)



energy density  $\frac{U}{V}$  is integrated area under curve.

4

(vi)

$$\frac{U}{V} = \int_0^{\infty} \frac{8\pi h \nu^3}{[\exp(h\nu/kT) - 1] c^3} d\nu$$

put  $y = \frac{h\nu}{kT}$  (1)

$$dy = \left(\frac{h}{kT}\right) d\nu$$
 (1)

$$\nu = y \frac{kT}{h}$$

$$\frac{U}{V} = \int_0^{\infty} \frac{8\pi h \cdot y^3 \cdot \left(\frac{kT}{h}\right)^3 \cdot \left(\frac{kT}{h}\right) dy}{c^3 [\exp(y) - 1]}$$

$$= \frac{8\pi h}{c^3} \left(\frac{kT}{h}\right)^4 \cdot \int_0^{\infty} \frac{y^3 dy}{[\exp(y) - 1]}$$
 (1)

$$= \frac{8\pi h}{(hc)^3} \cdot \frac{\pi^4}{15} = \frac{8\pi^5 (kT)^4}{15 (hc)^3}$$
 (1) 4

(vii) At  $T = 1000\text{ K}$ 

$$\frac{U}{V} = \frac{8\pi^5 (1.38 \times 10^{-23} \times 10^3)^4}{15 (6.63 \times 10^{-34} \times 3 \times 10^8)^3} = 7.60 \times 10^{-4} \text{ J m}^{-3}$$
 2

(viii)  $\left(\frac{U}{V}\right) \propto T^4$ 

$$\frac{\left(\frac{U}{V}\right)_T}{\left(\frac{U}{V}\right)_{1000}} = 10 = \frac{T^4}{(1000)^4} \quad T^4 = 10^{13} \quad T = 1778 \text{ K}$$
 2

### 3a. Superconductivity.

- (i) Basic features — due to boson condensation of current carriers<sup>(1)</sup>
- bosons are electron pairs<sup>(1)</sup>
  - energy gap between ground + excited states.<sup>(1)</sup>
  - so collisions do not excite pairs — so no resistance.<sup>(1)</sup>
  - pairs (Cooper pairs) have  $L=S=0$ .<sup>(1)</sup> 4

- (ii) Current carriers in normal phase are single electrons<sup>(1)</sup>
- In superconducting phase are electron pairs<sup>(1)</sup> with  $L=0$   $S=0$  ↑ 2

- (iii) Critical temperature occurs when  $kT$  breaks pair bonding<sup>(1)</sup>
- Critical field occurs when applied field action on electrons in pair breaks pair bonding.<sup>(1)</sup>
- One pair bond broken — individual electrons<sup>(1)</sup> are fermions — cannot condense into superconducting state. 3

- (iv) Isotope effect.

Seen exp'tly that  $T_c \propto \frac{1}{\sqrt{m}}$  <sup>(1)</sup>

Arises from lattice excitation where  $\omega_D = \sqrt{\frac{k}{m}}$ . <sup>(1)</sup>

Shows that lattice plays role in pairing of electrons to form Cooper pairs <sup>(1)</sup> 3

- (v) Cu has smaller resistivity than normal phase of superconductor Pb. <sup>(1)</sup>

This because interaction of single electron with lattice is smaller in Cu — good for normal conductivity. <sup>(1)</sup>

In Pb — larger interaction lattice-electron causes more scattering — higher resistivity in normal phase but also pairing to give rise to superconductivity <sup>(1)</sup> 3

(vi) Nature of high  $T_c$

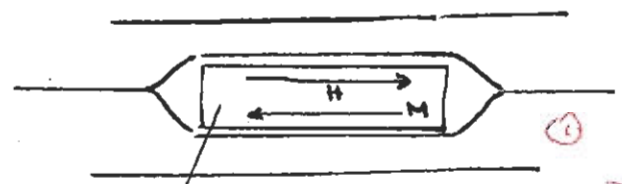
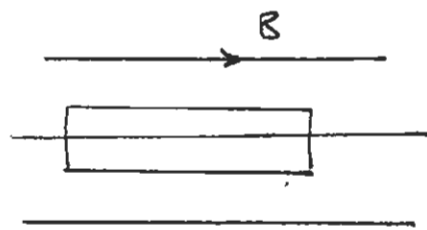
Ceramics - planes of Cu-O<sup>①</sup> spaced apart by other rows  
eg  $\text{YBa}_2\text{Cu}_3\text{O}_{7-x}$ . <sup>①</sup>

Cu is in mixed valence state - allows hole-pairs to form and move on Cu-O planes. <sup>②</sup>

Pairing mechanism not yet fully understood. <sup>3</sup>

(vii) Meissner effect.

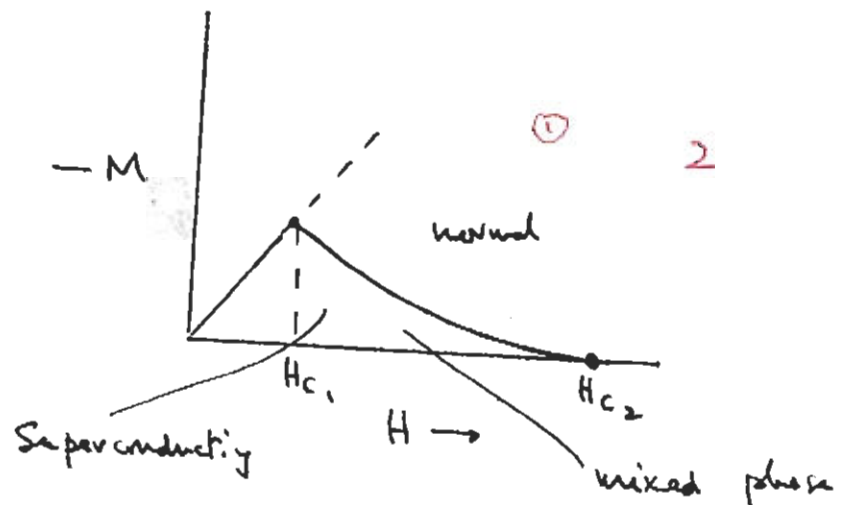
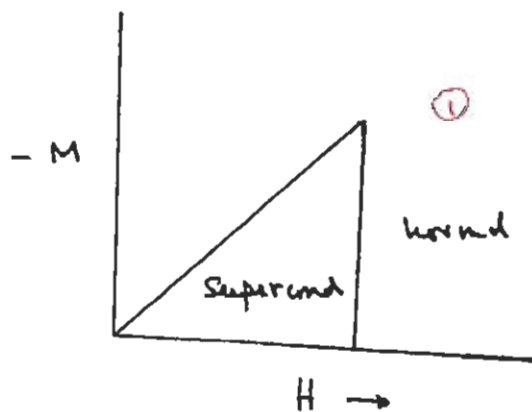
For sample in external magnetic field - when sample goes Superconducting - all flux of  $B$  expelled



Superconducting  
In sample  $B=0$   
Since  $M = -H$   
and  $B = \mu_0(H + M)$ .

(viii)

Type I

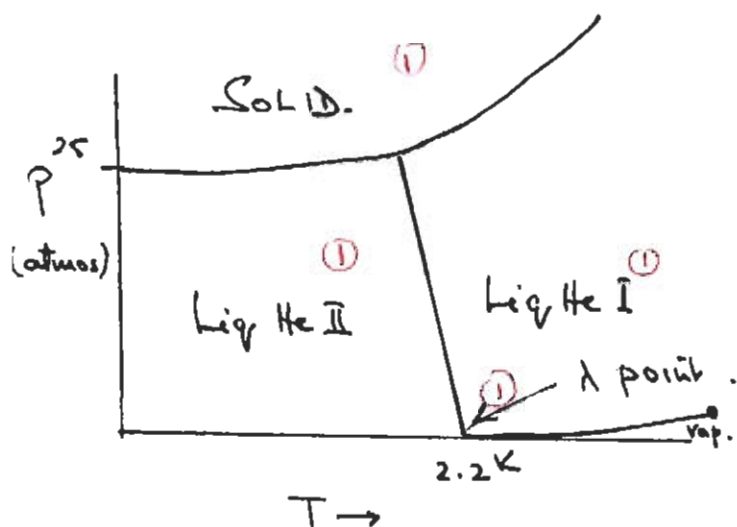


(ix) Examples. Type I

~~Al~~ - elements Nb, Pb, Al <sup>①</sup>

Type II alloys  $\text{Nb}_3\text{Sn}$  or high  $T_c$   $\text{YBa}_2\text{Cu}_3\text{O}_{7-x}$ . <sup>①</sup> <sup>2</sup>

3b.  $\text{He}^4$



Liquid  $\text{He}^4$  is considered by 2 fluid model

- ① 2 fluid — normal
- ① — Superfluid

- ① changes proportions — all superfluid at 0K
- ① all normal at  $\lambda$  point (2.2K)

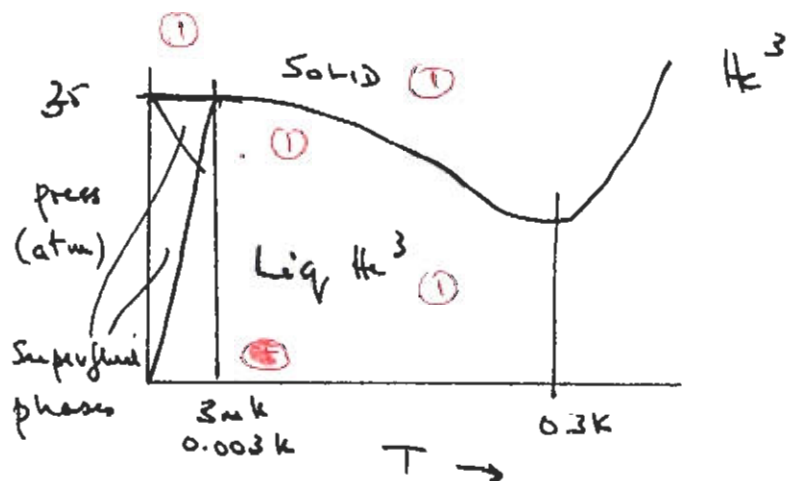
- ① Superfluid ( $\text{He}^4$  condensate) carries no entropy
- ① has no viscosity.

Viscosity measurements.

- ① Torsionally oscillating discs in liquid  $\text{He}^4$  — measured normal component attached to discs. — get high value for  $\eta$

- ① Flow thru narrow capillary — superfluid flows without viscosity — get only upper limit for  $\eta$ .

2 fluid model explains as measuring diff components of liquid  $\text{He}^4$ .



Liq  $\text{He}^3$  shows superfluid properties at  $T < 2\text{mK}$ . ①

This occurs because 2  $\text{He}^3$  atoms (fermions) can pair to form bosons that can then condense to ground state and become a superfluid phase. ①

Mechanism of pairing not well understood.

Phase diagram (superfluid part) is sensitive to magnetic field because pairs have  $S=1$  and  $L=1$ . — they are magnetic. 2

Different superfluid phases are differently magnetic and in applied field the phase that can interact with field to get to lowest energy increases — other phase decreases. 2

Low temperature.

— either Pomeranchuk cooling

or  $\text{He}^3/\text{He}^4$  dilution refrigerator



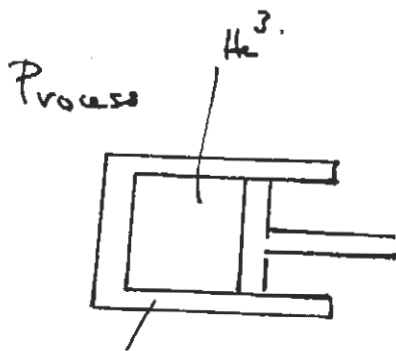
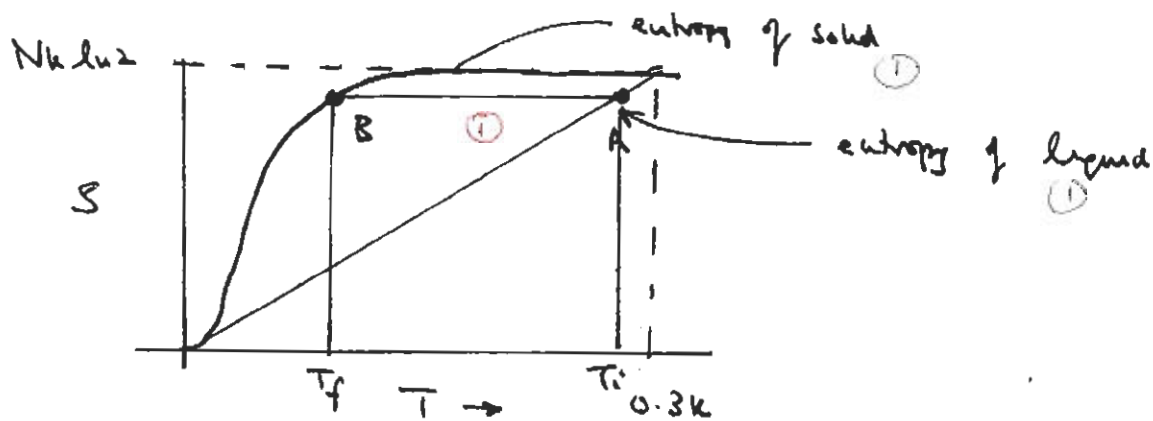
# EITHER

Pomeranchuk cooling.

Uses liquid/solid  $\text{He}^3$ . ①

Theory. In temp range  $0 \rightarrow 0.3\text{K}$  entropy  $S$  of solid  $\text{He}^3$  is greater than liquid  $\text{He}^3$ . ①

This because main entropy is in nuclear spins — this greater in Boltzmann distrib<sup>n</sup> in distinguishable atoms than in Fermi-Dirac distrib<sup>n</sup> in indistinguishable liq atoms. ①



- compress liq  $\text{He}^3 \rightarrow$  solid  $\text{He}^3$  adiabatically (const  $S$ ). ①

- this reduces temp  $T_i \rightarrow T_f$ . ①

Insulating walls

In practice start  $\sim$  ~~100~~ 0.2 K

and will end at  $\sim 10\text{ mK}$ . ①

One shot process. - connect sample to cell via conducting path ①

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OR

## Helium dilution refrigerator

- (i) Form of helium - mixture of  $\text{He}^3$  and  $\text{He}^4$  liquids
- (ii) Theory of cooling

Below triple point  $T = 0.96 \text{ K}$   $\text{He}^3$  and  $\text{He}^4$  liquids form a 2 phase system —  $\text{He}^3$  rich phase  
 $\text{He}^4$  rich phase

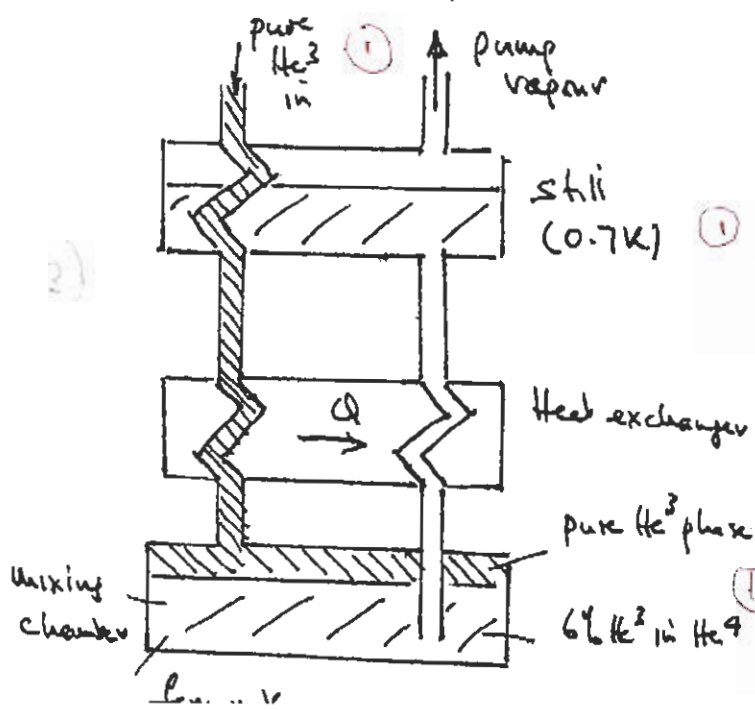
At  $T = 0$   $\text{He}^3$  rich phase  $\rightarrow 100\% \text{He}^3$   
 $\text{He}^4$  rich phase  $\rightarrow 6\% \text{He}^3 + 94\% \text{He}^4$

Cooling.

In mixing chamber (see diagram) phase of  $\text{He}^3$  floats on top of  $\text{He}^4$  ( $6\% \text{He}^3$ ) phase.

Cooling occurs at phase boundary as  $\text{He}^3$  atoms evaporate from  $\text{He}^3$  rich to  $\text{He}^4$  rich phases. This takes energy from  $\text{He}^3$  rich phase and cools it.

(iii) Schematic diagram



(iv) Experimental process.

Apparatus works by allowing above evaporation to occur continuously. (in cycle)

Dilute phase pumped — pure  $\text{He}^3$  gas comes off and is circulated down then heat exchanger to pure  $\text{He}^3$  layer in mixing chamber — continuous process.

(v) Still in operation